



Deseasonalisation of hydrological time series through the normal quantile transform

Alberto Montanari*

Faculty of Engineering, University of Bologna, Via del Risorgimento 2, 40136 Bologna, Italy

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Abstract

A procedure for deseasonalising hydrological time series is presented which is based on applying the normal quantile transform (NQT). The scientific literature recently presented examples of utilisation of the NQT that makes the cumulative distribution function (CDF) of the transformed hydrological variable Gaussian. The technique proposed here assumes that the CDFs of variables that form a seasonal time series are non-stationary and periodic; hence the NQT formulation is allowed to be periodic as well. The year is divided into a number of non-overlapping periods, each one characterised by an individual expression for the NQT. The latter is estimated empirically by applying a robust approach that avoids abrupt changes of the transformation through time. The resulting transformed time series is characterised by a single, stationary CDF which is no longer affected by the periodicity. The proposed technique is applied to deseasonalise hydrographs of mean daily flows observed in Italy and the United States.

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1. Introduction

Generation of hydrological data is increasingly being used to produce long synthetic records that allow river engineering works and water management policies to be tested for many different scenarios. In recent years, for instance, generation of synthetic river flow was performed by many authors to study the vulnerability of bridges, to analyse the effects of land use change (Brath et al., 2003; Rosso and Rulli, 2002) and to infer the shape of the flood frequency

distribution (Blazkova and Beven, 1997; Cameron et al., 1999). Different techniques can be used to generate synthetic observations, the choice depending on the aim of the analysis and the statistical behaviour of the time series to be generated. One possible approach is based on the application of stationary stochastic processes. For instance autoregressive integrated moving average models (ARIMA) can be used to generate non-intermittent series (Box and Jenkins, 1976; Montanari et al., 1997). An alternative approach can be taken by using point processes like the Neyman–Scott and Bartlett–Lewis models, that can be applied to generate intermittent data (Cox and Isham, 1980). However, application of these

* Tel.: +39 051 2093356; fax: +39 051 331446.

E-mail address: alberto.montanari@unibo.it.

approaches to hydrological data observed at time step shorter than one year is complicated by the fact that these data are affected by seasonal non-stationarity caused by the annual cycle. Such seasonality cannot be reproduced using a stationary model. The seasonal non-stationarity translates into periodic variation of the cumulative distribution function (CDF) of the observed variables and, in particular, their mean, variance, covariance, and higher order moments.

Stochastic modelling of seasonal data can be performed by applying seasonal models or, alternatively, by transforming the given time series into a stationary one. In the second case, one performs the so called deseasonalisation. Once synthetic transformed data are generated by means of a stationary stochastic process, the inverse of the deseasonalisation is applied to obtain a synthetic realisation of the original, non-stationary time series.

Several deseasonalisation techniques can be found in the hydrological and statistical literature that are usually based on removing the periodicity in the mean and variance of the data (Kottegoda, 1980; Yevjevich, 1984; Cleveland et al., 1990; Salas, 1993). In many real world applications, this approach is sufficient to allow a satisfactory application of a stationary stochastic process. However, it is important to note that only mean and variance are deseasonalised by this type of transformation. Higher order moments may still be periodical. Furthermore, the estimates of the seasonal components of mean and variance are affected by significant uncertainties that increase with decreasing scales of aggregation of the data. Indeed, for short aggregation periods the hydrological data are often highly non-Gaussian and, therefore, more significantly affected by outliers and measurement errors.

This paper presents a technique for deseasonalising hydrological time series. The approach is aimed at removing the periodicity in the CDF of the data (not only in the mean and variance). The technique is based on the application of the normal quantile transform (NQT): the year is divided into an arbitrary number, M , of non-overlapping periods, each characterised by an individual expression of the NQT. Thus the data corresponding to each period are transformed so that they follow a Gaussian distribution.

The paper is organised as follows. The NQT and the proposed deseasonalisation technique

are described in the second and third sections, respectively. The fourth section reports two examples of the deseasonalisation technique to daily river flow records. The fifth section presents concluding remarks.

2. The normal quantile transform

After the CDF of an arbitrarily distributed variable is determined, the normal quantile transform (NQT) is a transformation that can be used to make this CDF Gaussian. The NQT is fully described by Kelly and Krzysztofowicz (1997) and Krzysztofowicz (1997). The NQT has been used in hydrological simulation studies (see, for instance, Moran, 1970; Hosking and Wallis, 1988), as well as in hydrological forecasting models (see, for instance, Krzysztofowicz and Kelly, 2000; Krzysztofowicz and Herr, 2001; Krzysztofowicz and Maranzano, 2004a,b) and in procedures identifying the dependence structure of non-stationary hydrological time series (Maranzano and Krzysztofowicz, 2004).

In order to make the terminology that will be used through the paper clear, let us suppose that the observations $s(t)$, $t=1, \dots, T$ of a hydrological time series of length T are outcomes from the population of an arbitrarily distributed random variable S . The CDF of S is a function that gives the probability of S taking on a value lower than or equal to any $s(t)$; that is,

$$F_S[s(t)] = \Pr[S \leq s(t)], \quad (1)$$

where \Pr denotes the probability. In the same way, the multivariate (k -variate) CDF of a series of k random variables $S(t), \dots, S(t+k-1)$ is a function such that

$$F_S^k[s(t), s(t+1), \dots, s(t+k-1)] = P[S(t) \leq s(t), S(t+1) \leq s(t+1), \dots, S(t+k-1) \leq s(t+k-1)], \quad (2)$$

where $1 \leq k \leq T$.

The composition of the inverse of the standard Gaussian CDF, Q^{-1} , and the CDF $F_S[s(t)]$ of the hydrological time series defines the NQT of the original observation $s(t)$,

$$Ns(t) = Q^{-1}\{F_S[s(t)]\}, \quad (3)$$

in which N indicates that the variables are in the transformed space.

A CDF $F_s[s(t)]$ has to be chosen. In this study the empirical NQT (Krzysztofowicz, 1997) was applied, therefore approximating $F_s[s(t)]$ with the corresponding sample frequency $\hat{F}_s[s(t)]$, which is estimated using a plotting position. Stedinger et al. (1993) list several plotting positions that can be used for this purpose. The choice of the optimal one is conditioned by the probability distribution of the data. In this study $\hat{F}_s[s(t)]$ was computed by applying the Weibull plotting position, which gives unbiased exceedance probabilities for all distributions (Stedinger et al., 1993). The Weibull plotting position reads

$$\hat{F}_s[s(t)] = \frac{j(t)}{n+1}, \quad (4)$$

in which $j(t)$ is the position occupied by $s(t)$ in the sample rearranged in ascending order.

In summary, the empirical NQT involves the following steps. (a) For each $s(t)$, compute the cumulative frequency $\hat{F}_s[s(t)]$. (b) For each $\hat{F}_s[s(t)]$, estimate the standard normal quantile, $Ns(t)$, and associate it with the corresponding $s(t)$. Thus, a discrete mapping of (3), which gives the NQT, is obtained for the observed range of data.

In order to be able to apply the inverse of the NQT for any $Ns(t) \in \mathcal{R}$, linear interpolation is used to connect the points of the discrete mapping previously obtained. The region beyond the minimum and maximum available $Ns(t)$ values is covered by fitting an extreme value probability distribution to the extreme data values of the $s(t)$ sample. Specifically, the 50 highest and 50 lowest values of $s(t)$ are extracted and fitted using Gumbel and Weibull distributions, respectively (Kottegoda and Rosso, 1997). In the event that a synthetic observation, $Ns^*(t)$, generated in the normalised space falls outside the range covered by the discrete mapping of (3), the inverse transformation of the NQT can be computed as

$$s^*(t) = P^{-1}\{Q[Ns^*(t)]\}, \quad (5)$$

where P^{-1} is the inverse of the fitted Gumbel (or Weibull) distribution.

Maranzano and Krzysztofowicz (2004) list the advantages of the empirical NQT. One of them is that

it is free of any distributional assumption and therefore it can be applied also when it is difficult to identify a suitable parametric model for the distribution of a hydrological variate.

3. Application of the normal quantile transform for deseasonalising hydrological time series

One relevant property of seasonal time series is the presence of periodicities in their statistics. An application of the NQT on a time series showing such seasonality will not eliminate these statistical periodicities because it uses a transformation that is stationary in time. Consequently, the transformed time series would remain non-stationary. In order to obtain a stationary time series, the NQT should instead be different in the different periods of the year and, thereby, generate a periodical NQT (PNQT). A technique is suggested here for performing such type of transformation that requires a partitioning of the year into intervals (periods). Application of the empirical NQT to non-stationary time series was also considered in the recent works by Krzysztofowicz and Herr (2001) and Krzysztofowicz and Maranzano (2004a,b).

In detail, the procedure is structured according to the following steps.

(a) The year is divided into an arbitrary number, M , of non-overlapping periods. The i th period, $i = 1, \dots, M$, will be denoted with the symbol R_i . M is to be chosen depending on the climatic conditions and in such a way as to ensure that within each period the data can be considered approximately stationary. In the following we will assume that the different periods have equal time length of k days. This assumption could be easily removed.

(b) A time window of suitable size, w_i days, is associated with each period R_i . It is such that $w_i > k$, and that the centre of the time window is coincident with the centre of the respective period R_i . The temporal window corresponding to the period R_i will be denoted with the symbol L_i . The condition $w_i > k$ implies that the time windows will be partially overlapping. The size of the time window is allowed to vary in such a way as to include more observed data in the periods characterised by a higher variability. In fact, since the empirical NQT is used (see Section 2),

it is advisable that transformations covering a larger range of observed data are estimated from a larger number of points. One must specify the maximum and minimum values, w_{\max} and w_{\min} , of the time window size. The actual width, w_i , of the window L_i will be determined according to a procedure described later.

(c) For each period R_i , all the data observed in the days comprised in the time window L_i , in the different years covered by the observed record, are collected. These data are used for the empirical NQT_i according to the procedure described in Section 2. Since all the data contained in the time window L_i are used to estimate the NQT_i , it follows that each observed data value is used for estimating one or more NQT_i belonging to neighbouring periods.

(d) The data contained in the period R_i are transformed by applying the NQT_i . Let $s_i(t)$ denote the observations contained in the time series of the hydrological data of period R_i , which are supposed to be outcomes of the random variable S_i . Then the PNQT can be written as:

$$Ns_i(t) = Q^{-1}[\hat{F}_{S_i}(s_i(t))] = Q^{-1}\left[\frac{j_{i,t}}{T_i + 1}\right], \quad \forall s_i(t) \text{ from } R_i, \quad \forall i \in [1, M], \quad (6)$$

where $j_{i,t}$ is the position occupied by $s_i(t)$ in the sample for the time window L_i that has been rearranged in ascending order; and T_i denotes the size of the sample for the time window L_i . Notice how the data contained in L_i are utilised to estimate the empirical NQT_i , which is used to transform the observations contained in R_i only.

As mentioned above, w_i is allowed to vary between w_{\min} and w_{\max} , depending on the variability of the data contained in each period. Thus, w_i is determined before estimating the NQT_i through the following steps.

The data belonging to each temporal window, L_i , $i = 1, \dots, M$ are collected by first assuming $w_i = w_{\min}$. The range, y_i , of the data contained in each L_i is computed. Let $s_i^*(t)$ denote the observations contained in the time series of the hydrological data of window L_i . Then y_i can be computed as

$$y_i = \max[s_i^*(t)] - \min[s_i^*(t)]. \quad (7)$$

$y_{\max} = \max(y_i)$ and $y_{\min} = \min(y_i)$ are evaluated.

The width of the time window, L_i , formerly equal to w_i , is updated to the new value w_i^* according to the linear increase of the width between w_{\min} and w_{\max} that depends on y_i as follows:

$$w_i^* = w_{\min} + \frac{w_{\max} - w_{\min}}{y_{\max} - y_{\min}}(y_i - y_{\min}). \quad (8)$$

This procedure assures that w_{\min} and w_{\max} are assigned to the windows in which the ranges are equal to y_{\min} and y_{\max} , respectively. In fact, one can easily see that $w_i^* = w_{\min}$ when $y_i = y_{\min}$ and $w_i^* = w_{\max}$ when $y_i = y_{\max}$. It is not necessary that w_i be an integer. When it assumes a real value, all the data collected in a number of days equal to the integer part of w_i will be included in the time window L_i , as well as a fraction of the observations collected in the two adjacent days equal to the fractional part of w_i . For instance, let us suppose that $M = 365$ and therefore $k = 1$ day (as in the application presented in Section 4), $i = 10$ and w_{10} results equal to 3.5. Then, time window L_{10} will include observations collected in days 9, 10 and 11, as well as 25% of data collected in day 8 and 25% of those of day 12. These data included in time window L_{10} will be used to estimate the NQT_{10} that will be applied to transform the data of period R_{10} ; namely, the data observed in the day 10 in the different years covered by the observed record. This procedure implies that the number of observations contained in each L_i is different for different values of the range y_i .

As mentioned before, the condition $w_i > k$ implies that the empirical NQT_i of period R_i is estimated on a larger data set than that in R_i , by including observations from adjacent periods (as it was illustrated in the example given above). This procedure induces some important consequences. First, because the estimation of the NQT_i is performed on the basis of a larger data set, the robustness of the approach is augmented. Second, the temporal variability of the NQT structure is more gradual through time. Third, the fact that L_i is longer than R_i has a potential drawback: because the data are non-stationary in the presence of seasonality, the empirical NQT constructed in this manner does not guarantee that the transformed data from R_i follow the standard normal distribution, as it would if $w_i = k$. Therefore, the PNQT requires a proper validation that can be carried out by using a test for normality with the data

from each period R_i , as it is done in the application presented in Section 4.

A proper choice of the lower and upper values, w_{\min} and w_{\max} , of the time window size is based on the following remarks. On the one hand, low values of w_i increase the efficiency of the deseasonalisation procedure. In fact, because the stationarity assumption within L_i becomes more justified, reducing the width of the time window L_i increases the likelihood of the transformed data from R_i to be Gaussian. On the other hand, small values of L_i may cause the empirical NQT_i to be significantly different in adjacent periods. This is a direct consequence of sampling effects that are amplified when estimating the NQT from small data samples. Abrupt changes of the NQT are not realistic from a physical point of view and, therefore, should be avoided. Hence, it follows that w_{\min} and w_{\max} should instead be selected by looking at the results of the PNQT validation and by looking at the autocorrelation function of the transformed series $Ns(t)$. This latter should confirm the absence of significant periodicity. Also, one should impose a minimum number of observations required for estimating the empirical NQT. Such minimum number depends on the variability of the data. As a purely empirical indication, one may refer to the application presented in Section 4 in which a daily time series is analysed and $M=365$ periods were used. Each of these periods represented a day of the year (thus $k=1$ day) while w_{\min} and w_{\max} were set to 20 and 30 days, respectively. One should note that different river flow regimes may require different choices.

The deseasonalised time series $Ns(t)$ is built by assembling blocks of data which should follow the standard normal distribution. Therefore, the probability distribution of $Ns(t)$ should be close to Gaussian. However, it should be noted that Gaussianity within each block is not guaranteed (see the above remark about the necessity to validate the PNQT) and, therefore, it is not guaranteed that the resulting deseasonalised series $Ns(t)$ is Gaussian. Moreover, one should not forget that the technique proposed here may not be able to deseasonalise the multivariate CDF of the series of k random variables $S(t), \dots, S(t+k-1)$ (see Section 2).

4. Application

The proposed deseasonalisation technique was applied to remove the seasonal periodicity in two long series of mean daily river flow. The first series refers to the mean daily flows of the Po River (Italy) observed at Pontelagoscuro, from January 1st, 1942, to December 31st, 1984, a total sample size of 15706 days. The Po River drains a large part of northern Italy. Its contributing basin has an area of about 70,000 km² while the mainstream length is about 652 km. Precipitation occurs mainly in spring and autumn, while during the summer there is a significant snowmelt effect in the upper parts of the watershed. The river flow regime is characterised by two high flow periods in autumn and late spring-early summer. Fig. 1 shows a plot of the observed record.

The second time series refers to the mean daily flows of the Potomac River (United States) at Little Falls, from January 1st, 1931 to December 31st, 1998, a total sample size of 24,837 days. The Potomac river drains a large basin with an area of 38,000 km², including part of the states of Maryland, Virginia, West Virginia, Pennsylvania and the District of Columbia. The mainstream length is about 616 km. The river flow regime is characterised by one high flow period in spring. Fig. 2 shows a plot of the observed series. The autocorrelation functions of the two series are shown in Figs. 3 and 4. A significant periodicity is present in the data at the annual lag (i.e. lag = 365 days).

Observations collected on February 29th of leap years were removed from the records in order to keep

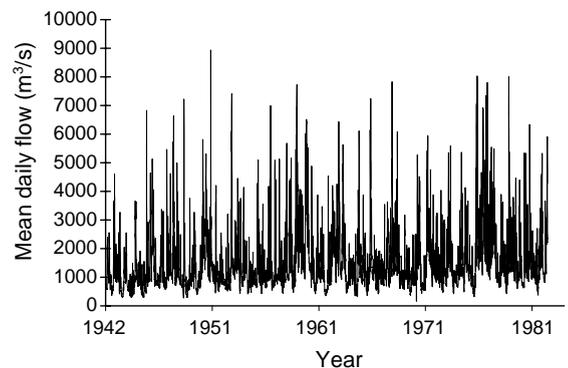


Fig. 1. Plot of the Po River mean daily flow original series (1942–1984).

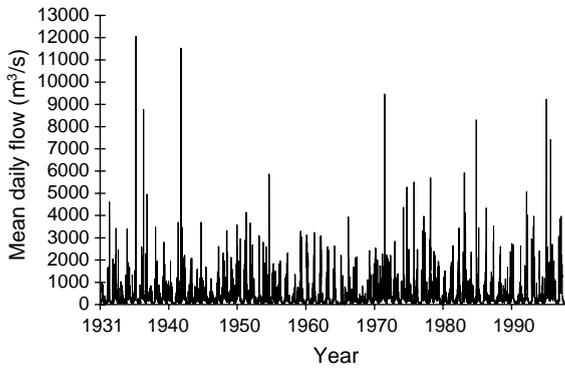


Fig. 2. Plot of the Potomac River mean daily flow original series (1931–1998).

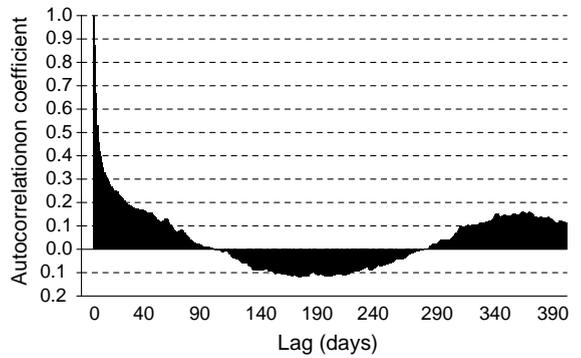


Fig. 4. Autocorrelation function of the Potomac River mean daily flow original series (1931–1998).

the period of the seasonality unaltered. The PNQT transformation was applied by selecting a number $M=365$ of periods, one for each day of the year. It follows that $k=1$ day. w_{\min} and w_{\max} were fixed at 20 and 30 days, respectively. These choices assured that, in the case of the Po River series, which was the shortest record analysed here, the NQT_i were estimated from a minimum of 860 observations for the period characterised by the lowest variability. Moreover, a temporal window of at least 20 days assured the absence of abrupt changes of the NQT in time.

The effectiveness of the PNQT to make the data from each period R_i Gaussian was checked by applying the Kolmogorov–Smirnov and probability plot correlation coefficient normality tests. Note that

the data in each R_i should follow a standard normal distribution. Therefore, the distribution parameters were known a priori and were not estimated from the data. Both tests were always satisfied at the 95% confidence level. Figs. 5 and 6, referring to Po River and Potomac River series, respectively, show the results of the Kolmogorov–Smirnov test for each period R_i . The critical value is indicated with the dashed line.

Figs. 7 and 8 show the autocorrelation function of the resulting deseasonalised series. No appreciable periodicity is now present at lag 365, thus confirming the absence of significant seasonality at the yearly period.

In order to check the capability of the PNQT to make the transformed data close to Gaussian,

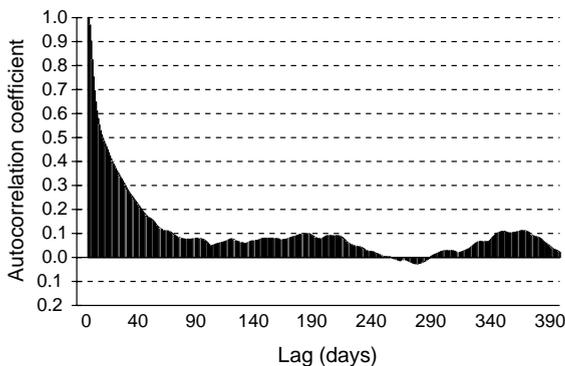


Fig. 3. Autocorrelation function of the Po River mean daily flow original series (1942–1984).

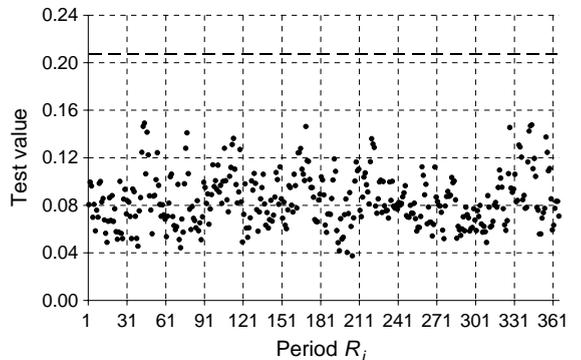


Fig. 5. Po River mean daily flow deseasonalised series (1942–1984). Kolmogorov–Smirnov normality test values for the data contained in each period R_i , $i=1, \dots, 365$. The dashed line indicates the critical value for the test statistic, at the 95% confidence level.

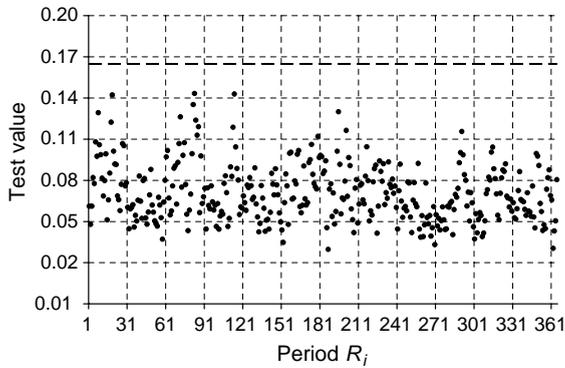


Fig. 6. Potomac River mean daily flow deseasonalised series (1931–1998). Kolmogorov–Smirnov normality test values for the data contained in each period R_i , $i = 1, \dots, 365$. The dashed line indicates the critical value for the test statistic, at the 95% confidence level.

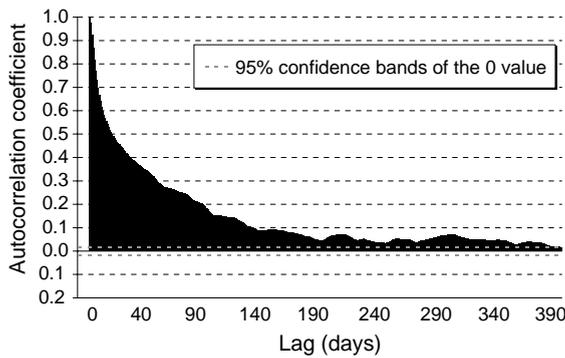


Fig. 7. Autocorrelation function of the Po River mean daily flow deseasonalised series (1942–1984).

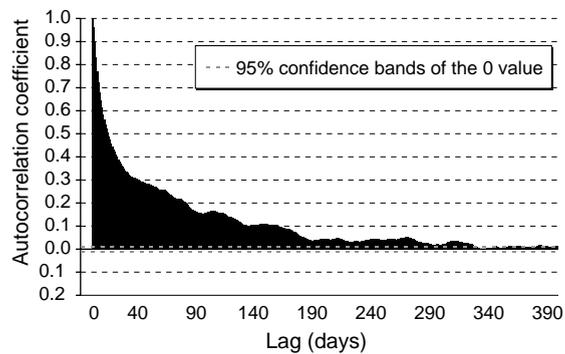


Fig. 8. Autocorrelation function of the Potomac River mean daily flow deseasonalised series (1931–1998).

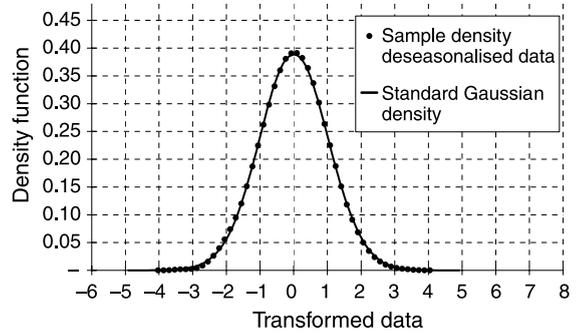


Fig. 9. Po River mean daily flow deseasonalised series (1942–1984). Comparison between sample probability density function and the standard Gaussian distribution.

Figs. 9 and 10 show the sample density functions of the two deseasonalised series compared with the standard normal density function. It can be seen that there is a satisfactory agreement. Fig. 11 shows a plot of the mapping given by (3) for the Potomac River data. This plot includes the periods corresponding to the first day of the months of January, March, May, July, September and November. Fig. 11 shows that the transformations are non-linear, that is, that the river flow data are non-Gaussian, as it was expected. A similar pattern was found by Krzysztofowicz (1997) when analysing flood crest data. Fig. 11 suggests that in the months from March through May the river flows are on average higher. However, extreme flows occur in many different periods.

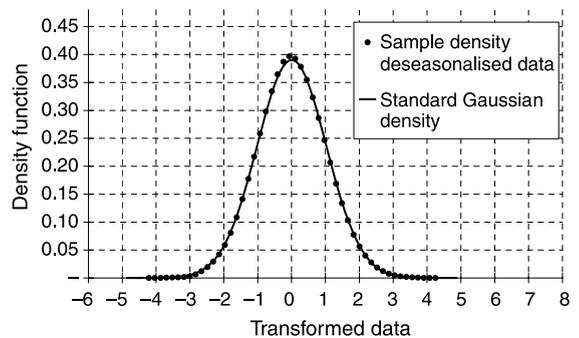


Fig. 10. Potomac River mean daily flow deseasonalised series (1931–1998). Comparison between sample probability density function and the standard Gaussian distribution.

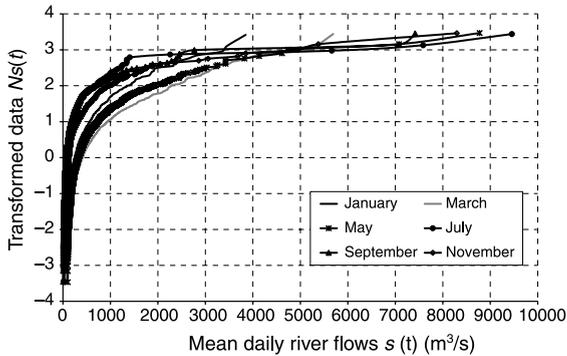


Fig. 11. Potomac River mean daily flow deseasonalised series (1931–1998). Plot of the mapping given by Eq. (3) for the first day of the months of January, March, May, July, September and November.

5. Concluding remarks

This paper presents a procedure for the deseasonalisation of hydrological time series that is based on the application of the normal quantile transform. One striking property of the proposed approach is its capability to deseasonalise the whole CDF of the data.

The proposed technique has been applied to two series of daily river flows observed in Italy and the United States, and the resulting deseasonalised data were found to be no longer affected by significant periodicity. Moreover, the proposed approach succeeded in bringing the CDF of each variable in the transformed series close to Gaussian. The paper provides details for inverting the proposed transformation; such operation is necessary in order to generate synthetic seasonal hydrological data by using stationary stochastic processes. Overall, the proposed deseasonalisation technique provides a valuable tool with which stationary stochastic models can be applied to seasonal hydrological time series.

A computer code (R programming language) for the application of the proposed technique is available for free download at the web page <http://www.costruzioni-idrauliche.ing.unibo.it/people/alberto>.

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