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Assessing the reliability of regional depth-duration-frequency equations for gaged and ungaged sites

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[1] Design storms are usually estimated by analyzing observed rainfall extremes. When there are no measured data for the location of interest, the design storm may be estimated using regional depth-duration-frequency equations (RDDFEs). The annual series of precipitation maxima obtained for a dense network of rain gages located in northern central Italy are analyzed in order to (1) investigate the statistical properties of the rainfall extremes and compare them with the characteristics reported in published studies, (2) develop RDDFEs for the estimation of design storms for storm durations from 1 to 24 hours and recurrence intervals up to 100 years in any location of the study area, and (3) assess the reliability of RDDFEs for ungaged locations through a comprehensive jackknife cross validation. *INDEX TERMS:* 1854 Hydrology: Precipitation (3354); 1821 Hydrology: Floods; 1869 Hydrology: Stochastic processes; 1833 Hydrology: Hydroclimatology; 9335 Information Related to Geographic Region: Europe; *KEYWORDS:* regional estimation, rainfall extremes, jackknife procedure, design storm estimation, northern central Italy

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1. Introduction

[2] Producing a reliable estimate of the design storm, herein defined as the rainfall depth expected at a specified location for a given storm duration and probability of occurrence, typically expressed in terms of return period, is essential to defining urban and rural planning strategies and to managing water resources. In addition, an estimation of the design storm is often needed when evaluating peak river flows by using conceptual rainfall-runoff models [*Brath and Rosso*, 1993]. This approach is frequently used to support the design of river engineering works when considering ungaged river basins.

[3] It is particularly difficult to estimate design storm when ungaged locations are considered. Hydrologists frequently address this problem by means of regional frequency analyses of rainfall extremes [*Buishand*, 1991], by pooling the rainfall information collected at several rain gages, either located within a fixed homogeneous geographical region [see for instance *Hosking and Wallis*, 1997; *Brath et al.*, 1998] or in a group of sites that are climatically similar without being necessarily located within a geographical region in the strict sense of the word [*Reed and Stewart*, 1989; *Burn*, 1990; *Schaefer*, 1990; *Faulkner*, 1999].

[4] A widespread regional approach is to develop empirical regional depth-duration-frequency equations (RDDFEs) expressing design storms as a function of storm durations and recurrence intervals [see, e.g., *Bell*, 1969; *Chen*, 1983]. Recent studies by *Alila* [2000] and *Madsen et al.* [2002] describe general frameworks from which regional relationships can be inferred. The former study proposes a general procedure for developing RDDFEs and shows an application for Canada. The latter study describes a generalized least squares (GLS) regression model for developing regional intensity-duration-frequency (IDF) relationships and shows an application to a relatively dense Danish gaging network.

[5] RDDFEs generally have a rather simple structure and make it easy therefore to estimate the design storm for ungaged locations. An example of an RDDFE for a wide geographical region of central Italy is provided by *Pagliara and Viti* [1993], following the approach proposed for India by *Kothyari and Garde* [1992].

[6] This study analyzes the annual series of precipitation maxima observed by a rather dense gaging network located in a wide geographical area of northern central Italy. The analysis has two main goals.

[7] First, the study investigates the statistical properties of the observed rainfall extremes in order to develop, by using a very well established methodology [*Alila*, 2000], simple RDDFEs to be used in any location of the study area for estimating the design storm for storm durations t ranging from 1 to 24 hours and recurrence intervals T varying from 2 to 100 years. The storm duration range was chosen in order to make the RDDFEs suitable for estimating the T-year storm for values of t comparable to the concentration times of the river basins belonging to the study region. The maximum recurrence interval was selected with the purpose of making the RDDFEs a viable tool to be used in the general context of the indirect estimation of the T-year flood for ungaged basins, with T up to 100 years. The centennial peak flow is a reference flood that is commonly employed when designing several kinds of river engineering works (e.g., levees of small to medium rivers).

[8] Second, since the regional equations are developed on a merely statistical basis and their application to ungaged sites might be considered inappropriate, the study develops

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a framework for assessing the reliability of RDDFEs for ungaged locations. In order to achieve this goal the study designs and applies an original jackknife cross-validation procedure [*Shao and Tu*, 1995], which is described in detail in section 5.

[9] The paper is structured as follows: Section 2 provides an overview of the procedure used for the formulation of RDDFEs; section 3 describes the study region and the local regime of rainfall extremes; section 4 outlines the RDDFEs developed for the study region by applying the procedure outlined in section 2; section 5 focuses on the assessment of the reliability of the RDDFEs for ungaged sites; section 6 covers the conclusions.

2. RDDFE Formulation Procedure

[10] The procedure adopted to develop RDDFEs is fairly similar to the procedure described by *Alila* [2000]; it involves several steps, which can be summarized as follows:

[11] 1. For each storm duration considered in the analysis (i.e., 1, 3, 6, 12, and 24 hours), use the annual maximum series of rainfall depths collected in the study region to fit an appropriate frequency distribution on an at-site basis.

[12] 2. Using the results of the at-site frequency analysis, compute the 2-, 5-, 10-, 20-, 50-, and 100-year design storms for each considered rain gage and storm duration.

[13] 3. Formulate, by means of linear regression analyses, regional depth-duration equations (RDDEs) of the following type:

$$R_{T,t} = A(t, T, \boldsymbol{\Theta})R_{T,24\mathrm{hr}} + B(t, T, \boldsymbol{\Theta}), \tag{1}$$

where $R_{T,t}$ and $R_{T,24\mathrm{hr}}$ are the T-year rainfall depths with t- and 24-hour storm duration respectively, while A and B are regional coefficients that may be considered as varying with the storm duration, recurrence interval, and geographical location, expressed in equation (1) by the vector $\boldsymbol{\Theta}$.

[14] 4. Formulate, using linear regression analyses and setting the intercept to zero [*Alila*, 2000], regional depth-frequency equations (RDFEs) defined as

$$R_{T,t} = R_{10\mathrm{yr},t}C(t, T, \boldsymbol{\Theta}), \tag{2}$$

where $R_{10\mathrm{yr},t}$ is the 10-year and t-hour design storm, while C is a regional coefficient that may depend on the storm duration, recurrence interval, and geographical location.

[15] 5. By combining equations (1) and (2), express $R_{T,t}$ as a function of the three above mentioned regional coefficients (i.e., A, B, and C) and the 10-year and 24-hour design storm through the following equation:

$$R_{T,t} = A(t, T, \boldsymbol{\Theta})R_{10\mathrm{yr},24\mathrm{hr}}C(24\mathrm{hr}, T, \boldsymbol{\Theta}) + B(t, T, \boldsymbol{\Theta}). \tag{3}$$

[16] Equation (3) may be applied to calculate the T-year t-hour design storm for any location within the region of interest, thus including ungaged sites, provided $R_{10\mathrm{yr},24\mathrm{hr}}$ has been estimated for that particular location.

[17] As specified in steps 1 and 2, at-site estimates of the design storm are used to formulate and develop the RDDFEs, while the procedure presented by *Alila* [2000] develops the regional equations through regression analyses from design storm estimates obtained for each rain gage by applying a regional model proposed by *Alila* [1999].

3. Study Region and Local Regime of Rainfall Extremes

[18] The study area includes the administrative regions of Emilia-Romagna and Marche, in northern central Italy, and occupies 37,200 km^2. The area is bounded by the Po River to the north, the Adriatic Sea to the east, and the divide of the Apennines to the southwest (see Figure 1). The northeastern portion of the study region is mainly flat, whereas the southwestern and coastal parts are predominantly hilly and mountainous.

[19] The mean annual precipitation (MAP) varies on the study region from about 500 to 2500 mm. Altitude is the factor that most affects the MAP, which exceeds 1500 mm starting from altitudes higher than 400 m above sea level (asl) and exhibits the highest values along the divide of the Apennines (see Figure 1).

[20] The available extreme rainfall data consist of the annual series of precipitation maxima with duration t equal to 1, 3, 6, 12, 24 hours and 1 day (i.e., from 9:00 A.M. to 9:00 A.M. of the following day) that were obtained for a rather dense network of recording (hourly rainfall) and observational-day (daily rainfall) rain gages from the National Hydrographic Service of Italy (SIMN). Table 1 summarizes the available rainfall data, whereas Figure 1 shows the location of the recording gages, along with a subset of 226 nonrecording gages for which recent observation of MAP values are available (i.e., 1950–1991).

[21] The storm duration range spans from the short and localized convective events to the long and large areal coverage storms associated with large cyclonic weather systems. A regional analysis of the dates of occurrence of short-duration rainfall extremes (i.e., t equal to $1 \div 3$ hours) showed remarkable consistency and a mean timing which virtually coincided with the beginning of August for the entire study area; the dates of occurrence of long-duration rainfall extremes (i.e., t equal to 24 hours or 1 day) showed, for the same area, less consistency and a mean timing which varied from the beginning of September to the beginning of November [*Castellarin and Brath*, 2002].

[22] Several regional frequency analyses of rainfall extremes were performed over the study area. These studies, according to the logic of the traditional "index flood" hypothesis [*Dalrymple*, 1960], proposed subdivisions of the region shown in Figure 1 into homogeneous climatic regions, within which the statistics of rainfall extremes for a given storm duration t are assumed to be constant [*Brath et al.*, 1998; *Brath and Franchini*, 1999; *Brath and Castellarin*, 2001]. This assumption contrasts with the findings of other studies, which demonstrated for different geographical areas and climatic contexts that the statistics of rainfall extremes vary systematically with location [*Schaefer*, 1990; *Alila*,

Figure 1. (opposite) Emilia-Romagna and Marche administrative regions; location of recording gages for different values of the minimum record length; Tyrrhenian Region; isoline maps of MAP and $R_{10\mathrm{yr},24\mathrm{hr}}$.

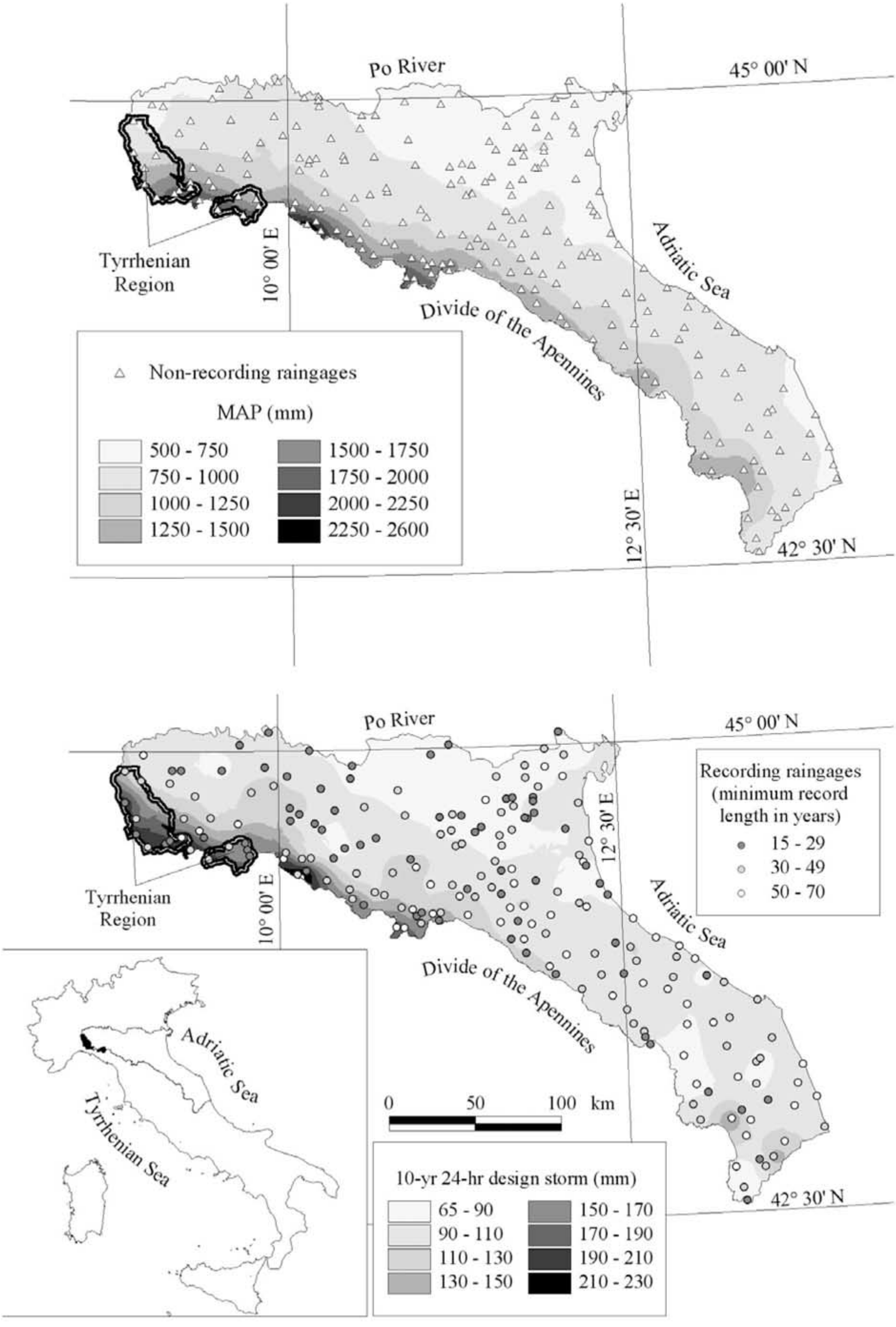
Po River
45° 00' N
Tyrrhenian Region
10° 00' E
12° 30' E
Adriatic Sea
Divide of the Apennines
42° 30' N
Non-recording raingages
MAP (mm)
500 - 750
750 - 1000
1000 - 1250
1250 - 1500
1500 - 1750
1750 - 2000
2000 - 2250
2250 - 2600
Po River
45° 00' N
Recording raingages (minimum record length in years)
15 - 29
30 - 49
50 - 70
Tyrrhenian Region
10° 00' E
12° 30' E
Adriatic Sea
Divide of the Apennines
Adriatic Sea
Tyrrhenian Sea
0 50 100 km
42° 30' N
10-yr 24-hr design storm (mm)
65 - 90
90 - 110
110 - 130
130 - 150
150 - 170
170 - 190
190 - 210
210 - 230

Table 1. Study Region: Number of Rain Gages and Station-Years of Annual Maximum Rainfall Data for Different Values of the Minimum Record Length (MRL)

	MRL = 15 years		MRL = 30 years	
	Number of Gages	Station-Years	Number of Gages	Station-Years
Recording rain gages	208	7876	132	6226
Observational-day rain gages	619	25904	419	21616

1999]. These studies also identified statistically significant relationships between these statistics and the MAP, which was used as a surrogate of geographical location. *Schaefer* [1990] and *Alila* [1999] showed, for two rather large geographical areas, that the coefficients of variation and skewness of rainfall extremes tend to decrease as the local value of MAP increases, and they proposed to enhance the index flood hypothesis for the regional frequency analysis of rainfall extremes by dispensing with the delineation of geographical areas and defining, instead, as climatically homogeneous subregions those areas which have a small MAP range.

[23] The applicability of the findings of *Schaefer* [1990] and *Alila* [1999] to the particular geographical and climatic context considered herein was investigated, and the available annual maximum series (AMS) of rainfall depth were analyzed, making use of a rain gage network with a higher resolution than the networks considered in the above mentioned studies. In particular, the variability of the sample L moment ratios [*Hosking*, 1990] of skewness (L-C_s) and variation (L-C_v) was examined against the variability of MAP.

[24] The results of the analysis show that the findings of *Schaefer* [1990] and *Alila* [1999] hold for a rather large portion of the study region, as the values of L-C_v and L-*Cs* of rainfall extremes are low in humid areas, while both statistics tend to increase, more or less markedly depending on the considered storm duration, as the local MAP value decreases. Nevertheless, the merger of two river basins indicated in Figure 1 as the Tyrrhenian Region, due to its closeness to the Tyrrhenian coast, exhibits a conflicting behavior. Table 2 reports the rainfall data for the Tyrrhenian Region. The region is mountainous and therefore humid (average MAP $\approx$ 1400 mm), yet it shows high L-*Cv* and L-*Cs* values for all the storm durations considered in this study. This anomaly is illustrated in Figure 2 for the annual maxima of daily rainfall. The figure shows (1) the sample L moment ratios against the MAP for 179 rain gages with at least 30 years of observations; (2) the weighted average MAP against the weighted average L-*Cv* and L-*Cs* for the Tyrrhenian Region; (3) the relationship between L-*Cv* and MAP identified by *Alila* [1999, Table 3, p. 31,650] for a storm duration of 24 hours; and (4) the moving weighted average curves of L-*Cv* and L-*Cs*, based on a moving window including 15 data points outside the Tyrrhenian Region (shaded curve). The average MAP, L-*Cv*, or L-*Cs* values are obtained by weighing each measure proportionally to the recorded length of the corresponding rain gage [see *Hosking and Wallis*, 1997]. Figure 2 shows the anomalous behavior of the Tyrrhenian Region and the consistency with the results obtained by *Schaefer* [1990] and *Alila* [1999] for the remainder of the region.

[25] The anomaly of the Tyrrhenian Region was already pointed out by *Castellarin et al.* [2001] and *Castellarin and Brath* [2002]. This atypical behavior may be accounted for by the proximity of the region to the Tyrrhenian shoreline and two windows in the Apennine divide, which locally drops below 1000 m asl, being normally above 1400–1500 m asl. These windows, known in literature as the Genoa gap, produce a fundamental topographic control, channeling the most severe disturbances coming from the south and originating over the Tyrrhenian Sea, and allowing them to have significant climatic control beyond the Apennine divide [*Tripoli et al.*, 2002]. The fact that the northern coastal area of the Tyrrhenian Sea exhibits a maritime rainfall regime and rather high coefficients of variation and skewness of rainfall extremes [*Brath and Rosso*, 1995] may partly explain the high L-*Cv* and L-*Cs* values observed for the Tyrrhenian Region, despite its rather high MAP values. These considerations were behind the formulation of the RDDFEs, which is described in the next section.

4. Formulation of the RDDFEs

[26] The formulation of the RDDFEs considered the 132 rain gages with hourly rainfall records longer than 30 years (see Table 1). The at-site estimates of the design storm for the 132 locations were calculated by taking the generalized extreme value (GEV) distribution as the parent distribution and by evaluating the distribution parameters through the L moments method [*Hosking*, 1990]. Several regional analyses showed that the GEV distribution is a suitable statistical model for representing the frequency distribution of rainfall extremes over the region considered herein [*Franchini and Galeati*, 1994; *Brath et al.*, 1998; *Brath and Franchini*,

Table 2. Tyrrhenian Region: Number of Rain Gages and Station-Years of Annual Maximum Rainfall Data for Different Values of the Minimum Record Length (MRL)

	MRL = 15 years		MRL = 30 years	
	Number of Gages	Station-Years	Number of Gages	Station-Years
Recording rain gages	10	343	7	283
Observational-day rain gages	34	1277	24	1024

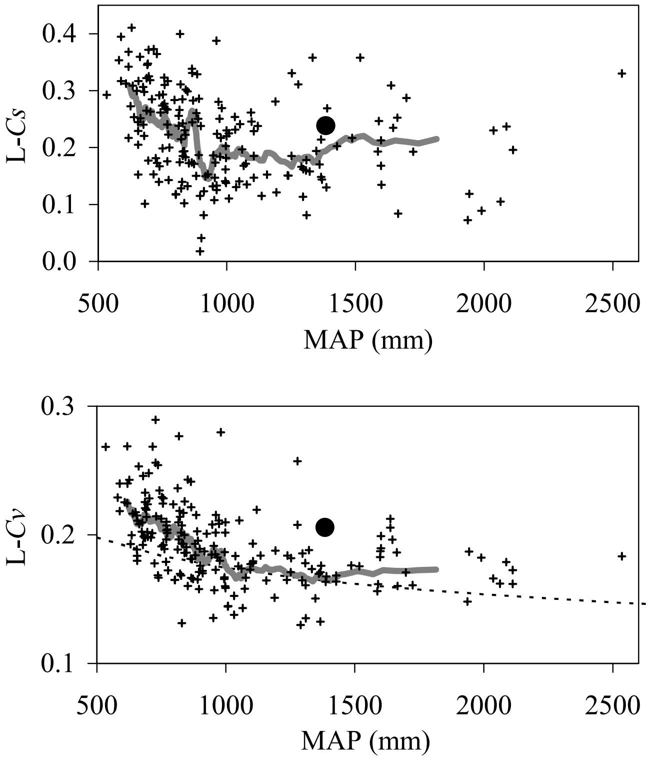

Figure 2. Annual maximum series (AMS) of daily rainfall: sample L moments versus mean annual precipitation (MAP) (plus signs); regional L moments versus regional MAP for the Tyrrhenian Region (solid dots); relationship between L-*Cv* and MAP identified by *Alila* [1999, Table 3, p. 31,650] (dashed line); moving weighted average curves of L moments for rain gages outside the Tyrrhenian Region (shaded curve).

1999]. The diagram of L moment ratios [*Hosking and Wallis*, 1993] reported in Figure 3 shows that the theoretical relationship between L skewness (L-*Cs*) and L kurtosis for the GEV distribution is very close to the regional L-*Cs* and L kurtosis values for the storm durations of interest.

[27] The GEV distribution is written as

$$F(x) = \exp\left\{-\left[1 - \frac{k(x-\xi)}{\alpha}\right]^{1/k}\right\}, \text{ for } k \neq 0, \tag{4}$$

$$F(x) = \exp\left\{-\exp\left[-\frac{(x-\xi)}{\alpha}\right]\right\}, \text{ for } k = 0, \tag{5}$$

where α, ξ, and k are the distribution parameters. As shown by equation (5), when $k = 0$ the GEV distribution is equal to the Gumbel distribution.

4.1. Regional Depth-Duration Equation

[28] The slope A and intercept B of equation (1) were initially assumed to be independent of geographical location; then the validity of this assumption was thoroughly analyzed, as described in this section. The two coefficients were calculated site by site for the chosen T (i.e., 2, 5, 10, 20, 50, and 100 years) and t (i.e., 1, 3, 6, 12, and 24 hours) values by a linear regression of the at-site estimates of rainfall quantiles (hereinafter referred to as S-GEV estimates). The analysis of the estimates of A obtained for T equal to 2, 5, 10, 20, 50, and 100 years and for t equal to 1, 3, 6, and 12 hours showed that A is heavily dependent on the storm duration t for a fixed value of the recurrence interval T. Further analyses focused on the dependence of A on T for a given t value and suggested that A should be taken as independent of T. First, the analyses considered t equal to 1 hour, calculated the average value, $\overline{A}$, of the six estimates of the coefficient A for the six chosen T values, and tested the null hypothesis H_0: $A = \overline{A}$ against the alternative hypothesis H_1: $A \neq \overline{A}$ at a 5% significance level [see, for example, *Chatfield*, 1983] for T equal to 2, 5, 10, 20, 50, and 100 years. Then the analyses considered in turn each one of the remaining storm durations (i.e., t = 3, 6, and 12 hours) and repeated the significance tests. The results indicated that the null hypothesis H_0 could not be rejected in anyone of the cases considered.

[29] Among several relationships investigated by the study, the following one-parameter power law was found to satisfactorily represent dependence between A and t:

$$A = a t^{-\frac{\ln(a)}{\ln(24)}}, \tag{6}$$

where a is the relation parameter and t is the storm duration in hours.

[30] The analysis of the different B values obtained for T equal to 2, 5, 10, 20, 50, and 100 years and for t equal to 1, 3, 6, and 12 hours showed that the coefficient is heavily dependent on both t and T. For instance, B values averaged over the six chosen T values are 30.1 mm for t = 1 hour, 24.6 mm for t = 3 hours, 17.5 mm for t = 6 hours, and 8.2 mm for t = 12 hours. Among several formulations tested in this context, the dependence of B on T and t was found to

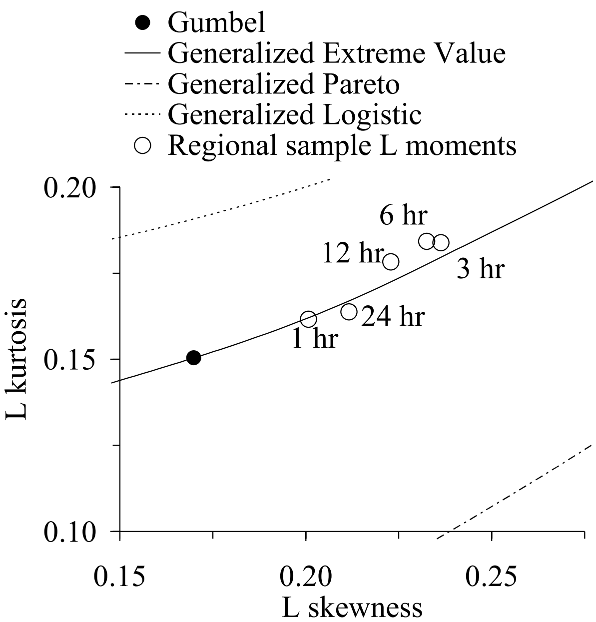

Figure 3. Diagram of L moment ratios for the application data.

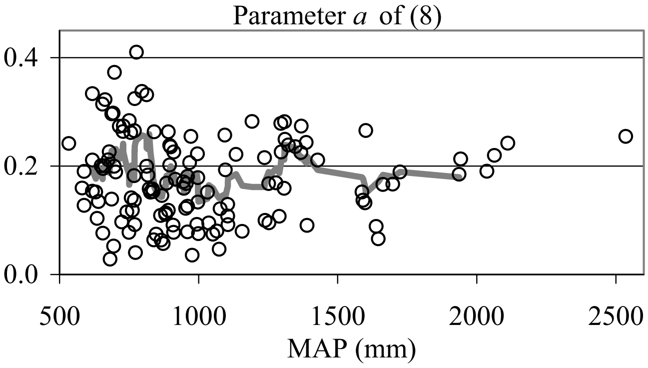

Figure 4. Variability of the at-site estimates of parameter a of equation (8) with MAP and moving average curve based on 11 data points.

be adequately expressed by the following three-parameter relation:

$$B = (24 - t)^b [c \ln(T) + d], \tag{7}$$

where b, c, and d are parameters, t is the storm duration in hours, and T the recurrence interval in years.

[31] By combining equations (6) and (7) into (1), we obtain the following relation:

$$R_{T,t} = a t^{-\frac{\ln(a)}{\ln(24)}} R_{T,24\mathrm{hr}} + (24 - t)^b [c \ln(T) + d]. \tag{8}$$

Independent of the values assigned to parameters a, b, c, and d, equation (8) produces $R_{T,24\mathrm{hr}}$ for a 24-hour storm duration, which meets therefore the S-GEV estimates of $R_{T,24\mathrm{hr}}$ when applied to gaged sites.

[32] Other studies, including *Alila* [2000], found that RDDEs depend on geographical location for storm durations larger than 60 min. In particular, *Alila* [2000] proposed two different RDDEs for Canada, depending on the local MAP value, used in lieu of geographical location; one relation holds for all sites with MAP $\leq$ 1200 mm, and a second relationship applies to all sites with MAP > 1200 mm. Therefore the assessment of the possible dependence of the parameters of equation (8) on geographical location represents a crucial step of the analysis. A first investigation of the possible dependence was performed as follows, the equation parameters a, b, c, and d were estimated for the entire study area by using a sequential quadratic programming (SQP) optimization procedure [*Gill et al.*, 1981] aimed at minimizing the differences between the RDDE and S-GEV estimates of $R_{T,t}$; then, by applying the same procedure, the RDDE parameters were reestimated separately for the Tyrrhenian Region and for the rest of the study area and, again, for the climatic regions reported by *Mennella* [1972] and the homogeneous rainfall regions delineated in the regional analyses of *Brath and Franchini* [1999] and *Brath and Castellarin* [2001]. The performances of the resulting RDDEs were compared in terms of overall adjusted R^2 and mean absolute relative error (MARE) against the S-GEV estimates. The RDDE developed for the whole study region (adjusted R^2 = 92.4%; MARE = 8.6%) was found to outperform the majority of regional RDDEs and to perform as efficiently as the regional relations in the other cases.

[33] Before assuming equation (8) to be independent of geographical location, a further analysis was performed. The study considered in turn each of the 132 rain gages and estimated the four RDDE parameters by minimizing the squared relative residuals between the S-GEV and RDDE estimates of the design storm for t = 1, 3, 6, and 12 hours and T = 2, 5, 10, 20, 50, and 100 years, as the trivial $R_{T,24\mathrm{hr}}$ cases could not be used. The a, b, c, and d estimates obtained for each site were then plotted against the local MAP value. Figure 4 illustrates the results of this analysis in connection with parameter a. The figure reports in particular the parameter estimates for all 132 sites and the moving average curve based on a moving window including 11 data points; the results obtained for parameters b, c, and d show similar features. Aside from a large variability caused by the limited amount of information used for the optimizations, the estimates of parameters a, b, c, and d did not exhibit any significant trend with MAP, thus making the approach adopted by *Alila* [2000] unsuitable for this context.

[34] Thus parameters a, b, c, and d of equation (8) were not considered as varying within the study region; their estimated values will be shown in conjunction with the RDDFEs in section 4.3.

4.2. Regional Depth-Frequency Equations

[35] Using linear regression analyses, the coefficient C of equation (2) was calculated for the reference T (i.e., 2, 5, 10, 20, 50, and 100 years) and t (i.e., 1, 3, 6, 12, and 24 hours) values. The estimates of C showed a marked dependence on the recurrence interval T, while the testing of hypothesis at a 5% significance level, similar to those outlined in section 4.2 for coefficient A, suggested that C can be considered independent of the storm duration t. A suitable description for the dependence on T can be provided by the following equation:

$$C = f \ln\left(\frac{T}{10}\right) + 1, \tag{9}$$

where f is a parameter and T is the recurrence interval in years. By combining equations (2) and (9), we obtain

$$R_{T,t} = R_{10yr,t}\left[f \ln\left(\frac{T}{10}\right) + 1\right]. \tag{10}$$

It is worth noting that for a recurrence interval of 10 years the application of equation (10) to gaged sites produces the S-GEV estimates of the design storm.

[36] The procedure described in the previous section was applied again to assess the dependence of f in equation (10) on geographical location. The results are consistent with the outcomes of the study by *Alila* [2000] and section 3 (see Figure 2). An accurate representation of the $R_{T,t}$ estimates (MARE = 5.4%, overall adjusted R^2 = 91.2%) can be obtained by using two different values for parameter f, a constant value, f_{TR}, within the Tyrrhenian Region (see Figure 1) and, for the remainder of the study region, a mathematical expression of f as a function of the local MAP value, which reads

$$f(\mathrm{MAP}) = g_1 - g_2 \ln(\mathrm{MAP}). \tag{11}$$

The f_{TR}, g_1, and g_2 values, all positive in order for equations (10) and (11) to make physical sense, will be shown in the next section. The minus sign in equation (11) is a direct consequence of the tendency of L-*Cv* and L-*Cs* to decrease for increasing MAP values, as observed for the study area outside the Tyrrhenian Region (see Figure 1).

4.3. Regional Depth-Duration-Frequency Equations

[37] By expressing $R_{T,24\mathrm{hr}}$ with equation (10) and replacing it in (8), we obtain the following expression for the RDDFE:

$$R_{T,t} = at^{-\frac{\ln(a)}{\ln(24)}} R_{10\mathrm{yr},24\mathrm{hr}} \left[f \ln\left(\frac{T}{10}\right) + 1 \right] + (24-t)^b [c \ln(T) + d] \tag{12}$$

where f is equal to f_{TR} for the Tyrrhenian Region and expressed through equation (11) as a function of MAP elsewhere. Once a suitable set of parameters a, b, c, d, f_{TR}, g_1, and g_2 is identified on the study area and a $R_{10\mathrm{yr},24\mathrm{hr}}$ estimate is available for the location of interest, along with a MAP estimate for sites outside the Tyrrhenian Region, equation (12) can be used to estimate the design storm $R_{T,t}$ for recurrence intervals up to 100 years and storm durations between 1 hour and 24 hours.

[38] Instead of using the a, b, c, and d values obtained from the identification of equation (8) and the f_{TR} and g_1 and g_2 values obtained while formulating equations (10) and (11), it was decided to recalibrate all parameters together (see section 5.3) through the SQP optimization procedure by minimizing differences in the S-GEV and RDDFE estimates of $R_{T,t}$. The RDDFE estimates of $R_{T,t}$ were computed for each site by utilizing the at-site estimate of $R_{10\mathrm{yr},24\mathrm{hr}}$. The optimization procedure led to the following RDDFEs:

$$R_{T,t} = 0.138 t^{0.624} R_{10\mathrm{yr},24\mathrm{hr}} \left[f \ln\left(\frac{T}{10}\right) + 1 \right] + (24-t)^{0.770} [0.474 \ln(T) + 0.951] \tag{13a}$$

where $R_{T,t}$ and $R_{10\mathrm{yr},24\mathrm{hr}}$ are expressed in mm, f is equal to f_{TR} = 0.259 within the Tyrrhenian Region and is expressed as

$$f = 0.602 - 0.055 \ln(\mathrm{MAP}) \tag{13b}$$

elsewhere, with MAP in mm (overall adjusted R^2 = 93.3%; overall MARE = 8.3%). It is interesting to note that according to equation (13b), f_{TR} = 0.259 corresponds to a MAP of around 500 mm, thus close to the minimum observed MAP for the study region and much smaller than the regional MAP of the Tyrrhenian Region (i.e., MAP $\approx$ 1400 mm). This outcome is consistent with the anomaly in the relationship between the L statistics of rainfall extremes and the MAP detected for the Tyrrhenian Region and reported in section 3.

5. Reliability Assessment of RDDFEs for Ungaged Sites

[39] Since one of the main interests of equation (13) is its applicability to ungaged locations, the analysis focused on assessing the reliability of design storm estimates produced by the regional equations for sites where rainfall data are not available. The reliability assessment was performed through an original jackknife resampling experiment [*Shao and Tu*, 1995].

5.1. Jackknife Procedure

[40] This procedure can be used to quantify the expected errors of the equations when applied to ungaged sites. Furthermore, this procedure quantitatively assesses the sensitivity of the parameters of equation (12) to hydrological information coming from each gaging station. Since the mathematical structure of (12) was identified on a mere statistical basis, the variability of the parameters a, b, c, d, f_{TR}, g_1, and g_2 associated with a disturbance of the hydrological information used during the calibration of the equations can provide useful indications on the robustness of the structure itself [*Brath et al.*, 2001]. The jackknife procedure can be summarized as follows:

[41] 1. Attention is focused on N_{30} = 132 gages with at least 30 years of observation of the hourly rainfall (Table 1).

[42] 2. In turn, one of these gaging stations, say, station i, is removed from the set of gages.

[43] 3. The parameters a, b, c, d, and f of equation (12) are reestimated through a SQP optimization procedure by using the information collected at the 131 remaining rain gages.

[44] 4. Using a suitable spatial interpolator, an isoline representation of $R_{10\mathrm{yr},24\mathrm{hr}}$ is generated from $N_{15} - 1$ at-site estimates, where N_{15} = 208 is the number of gages with at least 15 years of observation of rainfall (Table 1), and a jackknifed estimate of $R_{10\mathrm{yr},24\mathrm{hr}}$ for the location where station i stands is then retrieved from the isoline map. A 15-year-long record was deemed adequate for providing a reasonably accurate at-site estimate of the 10-year index storm, and the first runs of the procedure confirmed that the inclusion in the spatial interpolation phase of all rain gages with a record length from 15 to 29 years improves the reliability of the jackknifed estimates of $R_{10\mathrm{yr},24\mathrm{hr}}$ at the N_{30} gages.

[45] 5. A jackknifed estimate of the MAP for station i is retrieved from an isoline map generated from the 225 MAP values observed at the available recording gages, station i excluded (see Figure 1).

[46] 6. Using the jackknifed estimates of $R_{10\mathrm{yr},24\mathrm{hr}}$ and MAP and equation (12) with the recalibrated parameters, jackknifed estimates of $R_{T,t}$, with t = 1, 3, 6, 12, and 24 hours and T = 2, 5, 10, 20, 50, and 100 years, are computed for station i.

[47] 7. Steps 2–6 are repeated $N_{30} - 1$ times, considering in turn one of the remaining gages;

[48] 8. The $5 \times 6 \times N_{30}$ jackknifed estimates of $R_{T,t}$ are then compared with the corresponding S-GEV estimates.

[49] Step 4 mentions the use of a spatial interpolator. On applying the jackknife procedure, several spatial interpolators and options were tested, and the kriging interpolator with an exponential variogram [see, for example, *Delhomme*, 1978], working with up to eight neighboring rain gages located within a maximum search radius of 40 km from the site of interest, provided the best reproduction of the at-site estimates of $R_{10\mathrm{yr},24\mathrm{hr}}$ at both the N_{15} (bias = 2.2%; root mean square error = 14.1%) and N_{30} (bias = 1.4%; root mean square error = 13.0%) rain gages considered, and it was chosen therefore as the spatial interpolator indicated at step 4. The kriging interpolator with a linear variogram provided the best

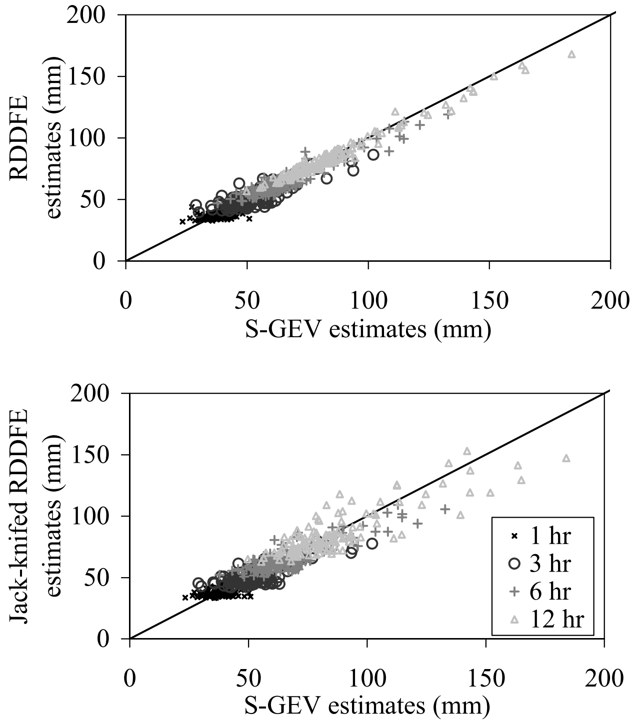

Figure 5. S-GEV versus RDDFE and jackknifed RDDFE estimates of the design storm for T = 10 years.

interpolation of the MAP values at N_{30} rain gages used to develop the RDDFEs (bias = −1.6%; root mean square error = 18.4%).

[50] Figure 5 illustrates how the application of the jackknife procedure affects the RDDFE estimates of the 10-year t-hour storm. The two panels plot the at-site estimates of the design storm against the corresponding RDDFE and jackknifed RDDFE estimates. The scatterplots show a visible reduction of the performance of the RDDFEs when the jackknife procedure is applied. Also, the overall adjusted R^2 of the jackknifed RDDFE estimates is equal to 86.2% and therefore appreciably lower than the 93.3% obtained for equation (13). Nevertheless, it has to be remembered that the jackknifed estimates shown in Figure 5 were computed without taking into account any of the data observed at the sites of interest, as if the locations were actually ungaged.

5.2. Analysis of the Results

[51] A quantitative analysis of the overall reliability of the jackknifed RDDFE estimates of the design storm used relative residuals evaluated as follows:

$$\varepsilon_{T,t,i} = \frac{\hat{R}_{T,t,i} - R_{T,t,i}}{R_{T,t,i}}, \quad (14)$$

where $\hat{R}_{T,t,i}$ is the jackknifed RDDFE estimate of the T-year t-hour design storm for a given site i, and $R_{T,t,i}$ is the corresponding S-GEV estimate.

[52] According to *Jakob et al.* [1999], because of sampling variability, a sample should contain at least $5T$ station-years of data to obtain accurate estimates of the T-year quantile. Hence the analysis of the residuals $\varepsilon_{T,t,i}$ between the jackknifed RDDFE and the S-GEV estimates for the N_{30} = 132 rain gages focused on recurrence intervals lower or equal to 10 years. The variability of the residuals with the MAP was carefully investigated in order to gain further information on the appropriateness of assuming that parameters a, b, c, and d of equation (8) are independent of geographical location (see section 4.1).

[53] Figure 6 illustrates some results of the jackknife cross validation, showing the residuals $\varepsilon_{T,t,i}$ against the local MAP value for t = 3 and 12 hours and T = 5 and 10 years, along with the moving average curves based on 11 data points. The analysis showed, for all storm durations and T = 2, 5, and 10 years, that 62% of the residuals are contained within a ±0.10 interval and fall outside the ±0.25 interval only in 4.9% of the cases. The diagrams of Figure 6, as well as those obtained for the remaining cases, do not show any systematic or significant variations with the MAP.

[54] Information on the reliability of the jackknifed RDDFE estimates of $R_{T,t}$ for longer recurrence intervals were obtained by analyzing the residuals for T = 50 years for the 52 recording gages with a sample length equal to or longer than 50 years (see Figure 1). Figure 7 shows the residuals for t = 3 and 12 hours. The analysis showed, for all storm durations and T = 50 years, that 61% of the residuals are contained within the ±0.10 interval and fall outside the ±0.25 interval in 10% of the cases, suggesting that the performance of the RDDFEs does not heavily depend on the T value considered.

[55] Figure 8 shows for a storm duration of 24 hours the 132 relative residuals for T = 10 years and the 52 residuals for T = 50 years, along with the corresponding moving average curves. Figure 8 introduces further elements for a correct interpretation of the magnitude of the residuals of the jackknifed RDDFEs. Because of the structure of equation (12), the residuals in Figure 8 for the 10-year recurrence interval do not depend on the jackknifed estimates of the equation parameters, namely, a, b, c, d, and f, and refer instead to the jackknifed estimates of $R_{10\mathrm{yr},24\mathrm{hr}}$, which were retrieved from the isoline maps. Also, because of the structure of equation (8), for t = 24 hours the residuals shown in Figure 8 for T = 50 years do not depend on the jackknifed estimates of the four parameters a, b, c, and d. The comparison of the diagrams in Figures 6, 7, and 8 shows that the order of magnitude of the residuals of the jackknifed RDDFEs for different storm durations, and recurrence intervals up to 50 years, is comparable with the order of magnitude of the residuals associated with the jackknifed estimates of $R_{10\mathrm{yr},24\mathrm{hr}}$ retrieved from isoline maps. Also, the patterns of the moving average curves in Figures 6 and 7 are very similar to the patterns of Figure 8, which are not ascribable to the assumption that the four parameters of equation (8) are independent of geographical location. This consideration further corroborates the results of the analysis of the variability of the at-site estimates of the parameters (see Figure 4), which led to the above mentioned assumption.

5.3. Considerations and Discussion

[56] As mentioned in section 4.3, the parameters of equation (12) were identified at the same time. The simultaneous estimation of a, b, c, d, and f improves the performances of the resulting RDDFEs for the available gaged sites but might reduce the robustness of the equations, thus reducing the reliability of the equations themselves when

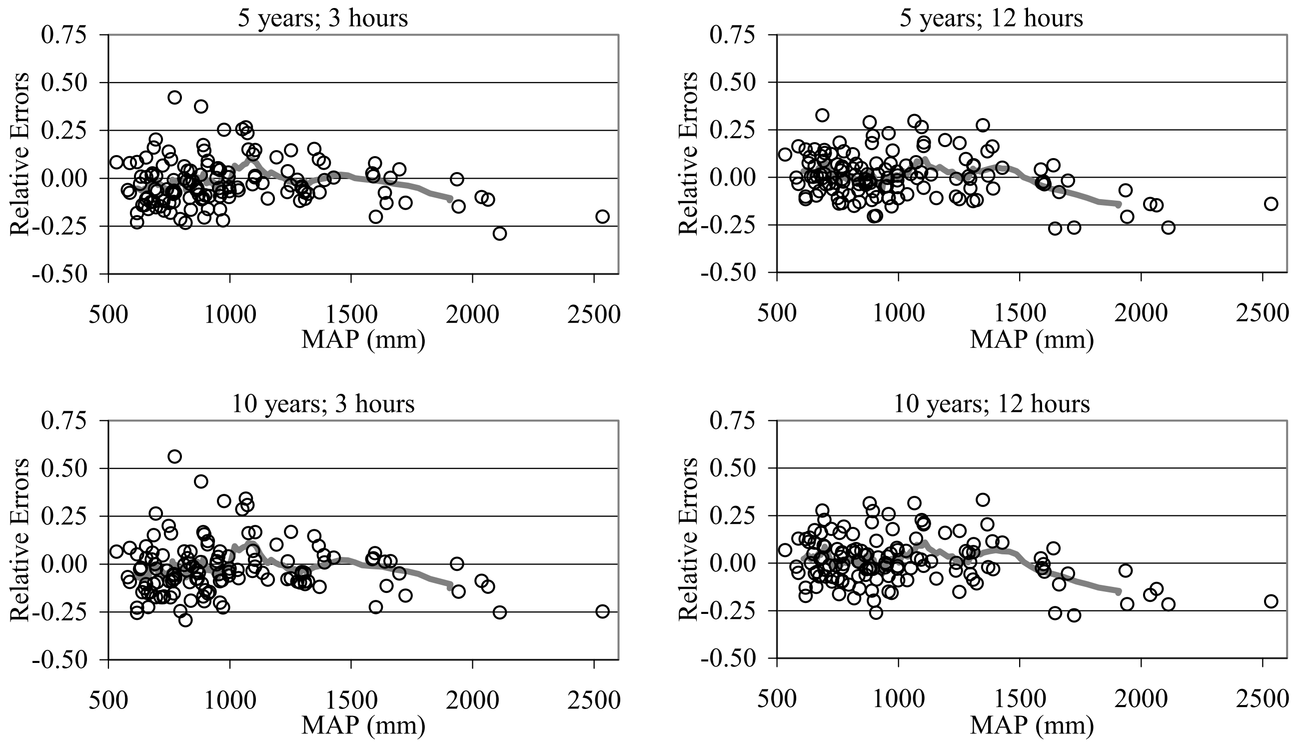

Figure 6. Relative residuals between jackknifed RDDFE and S-GEV estimates of the design storm for t = 3 and 12 hours and T = 5 and 10 years, and moving average curves including 11 data points.

applied to ungaged sites. This point was investigated through a series of jackknife simulation experiments similar to the experiment described in section 5.1. The results proved that the RDDFEs developed through the simultaneous calibration of all five parameters are more reliable than the equations obtained by combining RDDE and RDFEs for ungaged sites as well.

[57] The application of the jackknife procedure described in section 5.1 produced N_{30} = 132 estimates of parameters a, b, c, d, and f of equation (12), one set of parameters for each rain gage considered. The coefficient of variation, C_v, of the N_{30} estimates of the parameters is equal to 1.0% for a, 1.3% for b, and 3.6% for c and d. C_v is equal to 0.8% for f_{TR} and equal to 1.3% and 1.8% for g_1 and g_2, respectively. The 132 jackknifed parameters g_1 and g_2 showed a maximum relative variation from the mean parameter values of 20.6% and 32.9%, respectively. Nonetheless, the resulting relative variation of parameter f, as calculated from equation (11) through the jackknifed MAP estimates, never exceeded 10%. These considerations are indicators of the significant robustness of the mathematical structure of equation (12).

[58] Because of the differences existing in Mediterranean climates among the meteorological mechanisms responsible for the generation of annual maxima of hourly and daily rainfall, the first one typically convective and the latter usually connected to large cyclonic weather systems, one might argue that selecting a 24-hour design storm as the index storm in equation (12) or in (13) is not likely to make the regional equations suitable for short-duration rainstorms (i.e., $t = 1 \div 3$ hours). However, the assessment of the reliability of RDDFEs for ungaged sites showed that the performance was not heavily dependent on t (see Figures 6, 7, and 8). Furthermore, selecting $R_{10yr,24hr}$ as the index rainstorm has the advantage that the ratio between daily and 24-hour rainfall extremes shows only slight variations around the mean regional value of 0.89 for the whole study region [*Brath et al.*, 1998]. This provides a prompt and reliable estimation of $R_{10yr,24hr}$ for the observational-day (i.e., nonrecording) rain gages, which have a higher network density with respect to recording measuring gages (see Table 1).

[59] In order to have a clearer idea about the reliability of equation (13), let us refer to the estimation of the index storm for ungaged sites, which represents one of the most critical steps for the application of the regional procedures based on the "index flood" assumption to ungaged sites [*Brath et al.*, 2001]. Several studies take the mean of the annual rainfall maxima for the storm duration of interest t, M_t, as the index storm and suggest that it should be estimated at ungaged sites from isoline maps of annual rainfall maxima [*Schaefer*, 1990; *Alila*, 1999; *Brath and Castellarin*, 2001]. An iterative application of a resampling procedure analogous to steps 4 and 5 of the jackknife experiment described in section 5.1 produced N_{30} = 132 jackknifed estimates of M_t for t = 1, 3, 6, 12, and 24 hours using the kriging interpolator with a linear variogram. Figure 9 reports the relative residuals between the at-site and jackknifed estimates of M_t for t equal to 12 hours, but similar results hold for the remaining four storm durations. The top panel of Figure 9 considers all N_{30} = 132 rain gages, whereas the bottom panel focuses on the 52 stations with at least 50 years of observations of hourly rainfall. The relative residuals shown in Figure 9 are generally as large as or larger than residuals reported in Figures 6, 7, and 8. This consideration suggests that RDDFEs provide estimates of the design storm for ungaged locations that are as accurate as the estimates of M_t retrieved from isoline maps, at least for the region and spatial interpolators considered in this study.

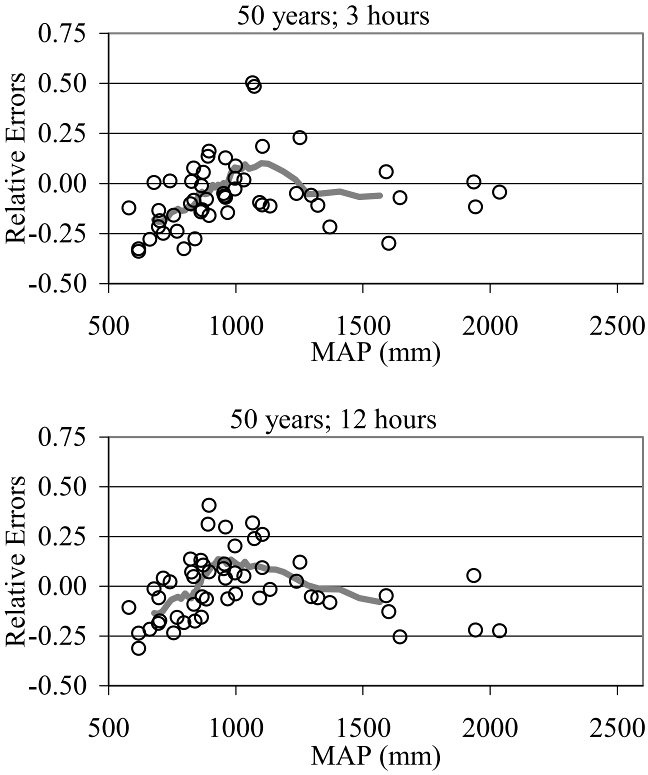

Figure 7. Relative residuals between jackknifed RDDFE and S-GEV estimates of the design storm for t = 3 and 12 hours and T = 50 years, and moving average curves including 11 data points.

[60] Practical applications of equation (12) should consider that the equations were developed through a statistical optimization procedure, and therefore they should meet the following restrictions: (1) The storm duration should not be shorter than 1 hour or longer than 24 hours; (2) the recurrence interval should not exceed 100 years; and (3) the site of interest must be located inside the study area (i.e., Italian administrative regions Emilia-Romagna and Marche). A careful application of RDDFEs should also consider that the regional equations were developed for rain gages located below 1500 m asl, while the study area can locally exceed 2000 m asl. Furthermore, the spatial interpolation of rainfall extremes or MAP adopted in the study is unable to reproduce microclimatic effects (e.g., rain shadow effects or differences existing between leeward and windward sides of the same mountain). In these circumstances the parameters of the regional equations might be locally more variable with geographical location than shown by parameter f of equation (12).

[61] Nonetheless, on the basis of the above considerations, the results of the analysis seem rather encouraging and indicate that RDDFEs are a viable and reliable alternative for a prompt estimation of the design storm, being particularly interesting for the design storm estimations at ungaged locations.

6. Conclusions

[62] The study (1) investigated the statistical properties of rainfall extremes for a wide region in northern central Italy developing, by using a very well established methodology, simple RDDFEs for the estimation of the rainfall depth for a given storm duration and recurrence interval (design storm) in any location of the study region, and (2) developed a framework for assessing the reliability of RDDFEs for ungaged locations through a comprehensive jackknife cross-validation procedure. The analysis showed that for a large portion of the study area, the statistical properties of rainfall extremes are consistent with the results provided by published studies referring to different geographical and climatic contexts. The analysis also detected a misbehaving region, in which, probably because of a topographic channeling of severe disturbances originated beyond the Apennine divide, the relationship between the statistics of rainfall extremes and mean annual precipitation contradicts the observations made for he reminder of the study area. This physical evidence was included in the RDDFEs developed by the study.

[63] The developed RDDFEs can be used for a prompt estimation of the T-year and t-hour rainfall depth in any location of the study region, for storm duration ranges t from 1 to 24 hours and recurrence intervals T up to 100 years. The only rainfall indexes required for the application of the RDDFEs are estimates of the 10-year and 24-hour rainfall depth and MAP, which can be evaluated for gaged locations from observed precipitation data, and retrieved from isoline maps for ungaged locations. The reliability of the RDDFEs was thoroughly investigated for their application to ungaged sites, which represents the most interesting use of regional equations. The investigation was

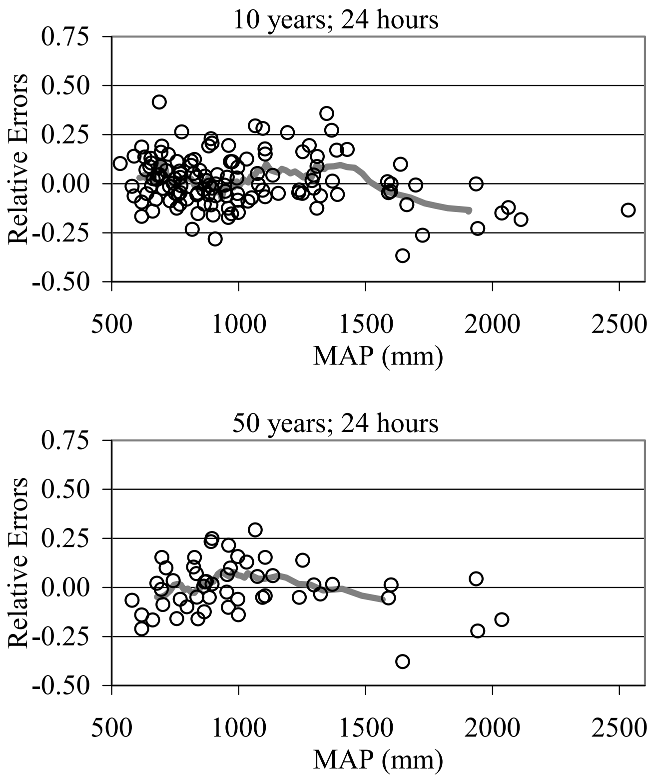

Figure 8. Relative residuals between jackknifed RDDFE and S-GEV estimates of the design storm for t = 24 hours and T = 10 and 50 years, and moving average curves including 11 data points.

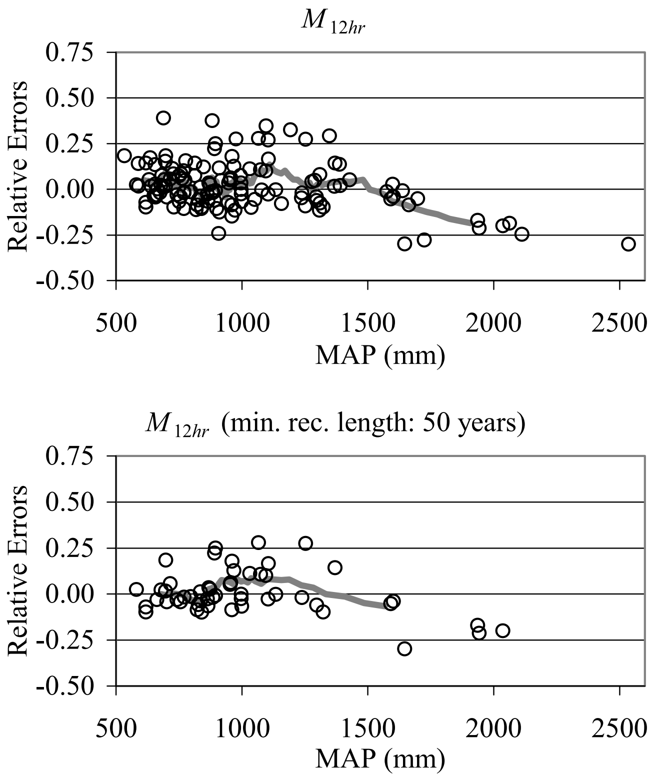

Figure 9. Relative residuals between the at-site and jackknifed estimates of M_{12hr} and moving average curves including 11 data points for all 132 rain gages and for the 52 rain gages with at least 50 years of observations.

performed by using a jackknife simulation experiment that provided further insight into the robustness of the mathematical structure of the proposed RDDFEs and the accuracy of the design-storm estimates calculated for ungaged sites. In particular, (1) the limited dependence of the parameters of the RDDFEs on pluviometric information coming from each rain gage proved that RDDFEs have a rather robust mathematical structure, and (2) the accuracy of RDDFE estimates of the design storms for ungaged locations showed a limited dependence on the recurrence interval and was comparable with the typical accuracy of the estimation of the mean annual rainfall maxima from an isoline map, which is a procedure suggested in several studies that advocate the use of regional models for estimating design storms at ungaged locations.

[64] The results of the analysis seem to prove that the proposed RDDFEs are practical and effective computational means for estimating design storms at any site of the study region. Future analyses focusing on different geographical regions and climatic conditions will provide useful indications about the general validity of regional equations.

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